

Wavelet Based Nonlinear Autoregressive Neural Network to Predict Daily Reservoir Inflow.

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Abstract

In spite of the ability of Artificial Neural Network (ANN) to handle nonlinear relationships in data, ANNs fail to predict with high accuracy in the presence of non-stationarity. Hydrological processes in nature exhibit non-stationarity due to many interrelated physical and other interrelated factors such as chaotic weather conditions. This paper presents the modelling of one such hydrological process, inflow, to the top most reservoir in the major cascaded reservoir system in Sri Lanka. This daily inflow series has been investigated to be nonlinear and nonstationary. Thus, the difficulties encountered in modelling the inflow series is addressed through a pre-processing strategy based on wavelet transform. Among the methods available in dealing with the nonstationary nature, the wavelet transform was used due to its ability to determine the frequency content of the signal and to assess and determine the temporal variation of this frequency content. The inflow series is decomposed into several sub-series using discrete wavelet transform (DWT). Consequently the appropriate sub-series resulted through the wavelet transform are used to model the original inflow using Nonlinear Autoregressive Artificial Neural Network with Exogenous Inputs (NAR-ANN). The results of the NAR-ANN with modified inputs are compared with the results of the base model i.e. NAR-ANN with raw inputs as well as a previously fitted cluster-based modular NAR-ANN. The results confirm the superiority of the wavelet-based approach over the other approaches, as it has the ability of capturing useful information on various resolution levels.

Keywords: wavelet, pre-processing, nonstationary, NARX, ANN

1. INTRODUCTION

Mahaweli hydropower system is the largest integrated rural development multi-purpose programme based on water resources in Sri Lanka. As such, accurate prediction of the generation of the hydroelectricity from the Mahaweli system is of vital importance to determine the ability of satisfying the future electricity needs and to minimize the wastage of resources. Forecasting the hydroelectric generation, however, requires a comprehensive knowledge of future availability of water resources. Modeling the inflow to the upmost reservoir Kotmale is substantial in that regard, as the entire system depends upon the changes in the natural behavior of this reservoir. A grid search based parameter tuning for the Nonlinear Autoregressive neural network (NAR-NN) was carried out previously to model the daily inflow in a univariate manner (Basnayake et al., 2017 a). It was identified that the fitted NAR-ANN does not adequately forecast inflow and thus be improved. The possible reasons for this less accurate forecast was noted to be the existence of extreme values in the inflow series along

with its non-stationarity. It was also proposed through Basnayake et al., (2017 b), to improve the performance of the model incorporating the cluster effect of inflow in order to address the existence of extreme values. This study presents a novel wavelet based application for the same purpose allied with a performance comparison between single NAR-ANN, cluster based NAR-ANN and wavelet based NAR-ANN. Modified codes of MATLAB Neural Network Toolbox and Wavelet Toolbox in Windows platform were used for model fitting.

2. RELATED LITERATURE

This study focuses in forecasting inflow to the upmost reservoir in Mahaweli river basin. It had been identified (Basnayake et al., 2017 a, Basnayake et al., 2017 b) that the inflow variable is nonlinear as well as nonstationary. Moreover the literature (Basnayake et al., 2017 b) reveals that Nonlinear Autoregressive with Exogenous Input (NARX) Artificial Neural Network (ANN), hereafter called as NARX-ANN, was considered a more suitable approach to investigate inflow. Although artificial neural network (ANN) methods have been used extensively as useful tools for prediction of hydrological variables, dealing with nonstationary data has many drawbacks (Cannas et al. 2006; Partal 2009). As one of the remedial measures to this problem, a hybrid modelling approach, namely the Wavelet analysis (WA) combined with ANN is discussed through literature. (Krishna, 2013)

WA is a much used tool for analyzing localized variations of power within a time series. By decomposing a time series into time–frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time. (Torrence and Compo, 1998). Wavelets, due to their inherent properties, have been explored for use in time series analysis. In addition, Wavelet Transform (WT) provides useful decompositions of original time series capturing useful information on various resolution levels. (Krishna, 2013). Continuous type of WT (CWT) determines both the scale contents of a signal and how they vary in time. Discrete wavelet transform (DWT), however, is generally used to decompose a series into sub-signals given proper wavelet and decomposition level. Consequently these sub-signals guide various time series analyses, such as wavelet decomposition, wavelet de-noising, wavelet aided complexity description, and wavelet aided hydrologic forecasting (Smith et al., 1998; Whitcher et al., 2002; Partal, 2009).

There are several applications of DWT in the field of hydrology with the objective of procurement of improved performance. Given below are some of those as an examples that can be extracted from the Literature.

- Rainfall–runoff models [Wang and Lee 1998, Labat et al. 1999, Coulibaly et al. (2000), Cannas et al. (2005) , Rahnama and Noury 2008, Nourani et al. (2011), Okkan (2012a)]
- Streamflow and reservoir inflow forecasting [Smith et al. 1998, Saco and Kumar 2000, Coulibaly et al. (2000), Anctil and Tape (2004), Kisi and Cimen (2011), Adamowski and Sun (2010), Okkan (2012b)]

All studies stated above discuss the idea of improving the performance of nonstationary time series through wavelet transform the efficiency of the DWT approach is confirmed and recommended. It is worth noting that the challenge in wavelet analysis is the selection of the mother wavelet function as well as the decomposition level of signal (Rafiee et al., 2009). Furthermore, the choice of the mother wavelet depends on the structure of the data to be analyzed. Literature had proven that many quantitative methods in finding the best mother wavelet function provide more accurate results than the qualitative methods, but no quantitative method is better than the other. Maximum energy to Shannon entropy ratio criterion [(Yan, 2007), (Kankar et al., 2011)] that is extensively used, is known to captures important information in the series (as combination of some other methods) and is suitable for discrete wavelet transform.

3. MATERIALS AND METHODS

Recall that the inflow series considered in this study is non-linear and non-stationary. Therefore, a wavelet based NAR-ANN approach is considered. Capturing the non-stationarity nature is achieved through DWT, which converts the main series into various sub series in a way that the useful information on various resolution levels are captured.

Given proper mother wavelet $\varphi(t)$ and decomposition level j , the DWT of a series $f(t)$ is operated as (Percival and Walden 2000):

$$w_{f((j,k))} = \int_{-x}^{+x} f(t) \psi_{j,k}^*(t) dt \text{ with } \psi_{j,k}(t) = a_0^{-j/2} \psi(a_0^{-j} t - b_0 k) \quad (1)$$

Where a_0 and b_0 are constants, integer k is time translation factor; $\psi^*(t)$ is the complex conjugate; $w_j(j, k)$ is the discrete wavelet coefficient under level j . In practice the dyadic (orthogonal) DWT in Eq. (2) is used commonly by assigning $a_0 = 2$ and $b_0 = 1$ (Daubechies 1992):

$$w_f(j, k) = \int_{-x}^{+x} f(t) \psi_{j,k}^*(t) dt \text{ with } \psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi(2^{-j} t - k) \quad (2)$$

The dyadic DWT allows for the decomposition of a series into a progression of “approximation” and “detail” coefficient sets under each level. In each but except the first stage only approximation coefficients are analyzed (Mallat 1989). Then, sub-signal $f_j(t)$ of original series under certain level j ($j = 1, 2, \dots, M, M$ is the biggest decomposition level) can be reconstructed by Eq. (3):

$$f_j(t) = \sum_k w_f(j, k) \psi^*(2^{-j} t - k) \quad (3)$$

These sub-signals may correspond to noise or deterministic components, and their sum equals original series. The wavelet used for dyadic DWT must meet the regularity condition with the regularity of order N (Mallat 1989):

$$\int_{-x}^{+x} t^k \psi(t) dt = 0, k = 1, \dots, N - 1 \quad (4)$$

In practice seven wavelet families can meet the Eq. (4), namely Haar, Daubechies (db N), Coiflets (coif N), Symlets (sym N), BiorSplines (bior $M.N$), and DMeyer (dmey) respectively (Chui 1992). Besides, the theoretically biggest decomposition level M in dyadic DWT is $\log_2(J)$ given the analyzed series has the length of J .

4. ANALYSIS

Wavelet based NAR-ANN was fitted to the daily inflow series obtained from 2013 to 2015 (3 years). Basnayake et al (2017 a) have identified the suitable time period to be 3 years, based on the lowest error in NAR-ANN model fitting through a lookback analysis for the same watershed. The first and the most crucial step was identifying the most appropriate mother wavelet function to carry out the wavelet transformation. As such, 125 mother wavelet functions [Haar, Daubechies (db:2-45), Symlets (sym:2-45), Coiflets (coif:1-5), BiorSplines (bior:1.1, 1.3, 1.5, 2.2, 2.4, 2.6, 2.8, 3.1, 3.3, 3.5, 3.7, 3.9, 4.4, 5.5, 6.8), Reverse BiorSplines (rbior:1.1, 1.3, 1.5, 2.2, 2.4, 2.6, 2.8, 3.1, 3.3, 3.5, 3.7, 3.9, 4.4, 5.5, 6.8) and DMeyer(dmey)] were tested to find out the best suited function for the inflow series of interest.

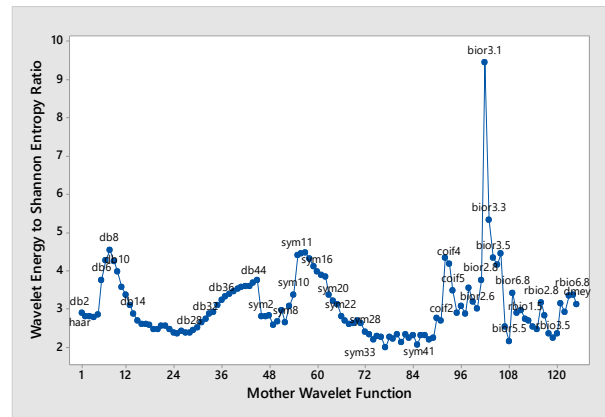


Figure 1: Wavelet energy to Shannon entropy ratios corresponding to DWT of inflow series using 125 mother wavelet functions.

As shown in Figure 2, there is a clear maximum for the “Bio Splines 3.1”. The same function was observed as maximum consistently throughout the inflow series for various time intervals, concluding that the most suitable mother wavelet function to transform the inflow is “Bio Splines 3.1”.

The main objective of this study is to test the effectiveness in wavelet transform to better capture the non-stationary nature of the inflow series in order to improve the forecast performance. This is achieved by initially transforming the original inflow series into various sub series. Consequently a NAR-ANN model is fitted to each sub-series separately and finally aggregate the results to produce the final output. As explained in the Literature, two key issues in DWT is the mother wavelet selection and the decomposition level choice. The maximum level of decomposition for a particular series is $\text{LOG}_2(N)$ where N being the size of the original series. Thus, for this study the maximum level is 10 i.e $\text{LOG}_2(1095)$.

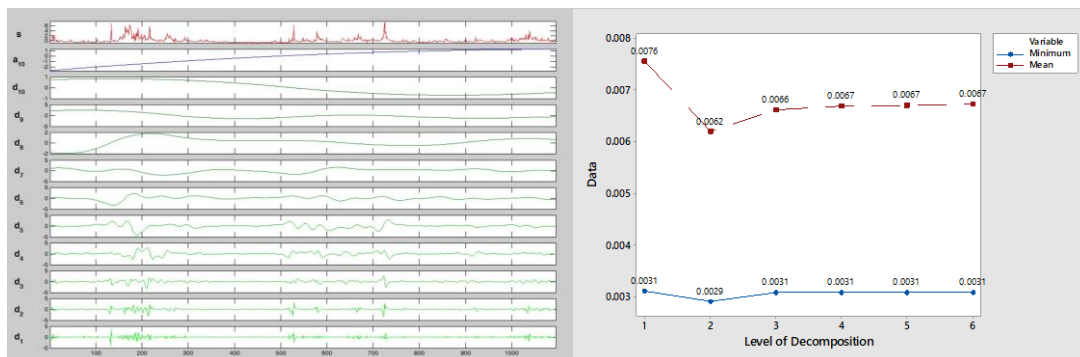


Figure 2: Wavelet decomposed series under maximum decomposition level (left), Minimum and mean grid search errors of wavelet NAR-ANN corresponding to levels 1 – 6.

Since NAR-ANN is a nonlinear modelling structure, fitting linear structures is of no use. As displayed in the Figure 2 (left), nonlinearities associated with the series a_{10} , d_7 - d_{10} are very much low. Therefore it was decided to fit NAR-ANN to the wavelet decompositions for levels 1 to 6 only.

The grid search based parameter tuning as stated by Basnayake et al. (2017 a) was carried out in fitting NAR-ANN and the resulted grid errors were analysed in order to identify the best level of decomposition. According to Figure 2 (right), level 2 of decomposition is resulted with both the minimum grid error and the mean grid error. However, after level 4, the errors seems to be consistent with the level of decomposition. Therefore, both levels 2 and 4 were tested in the forecasting, as the results may be different with pointwise errors than with the overall grid search error observed. Figure 3 below displays the graphical view of those two levels of decompositions.

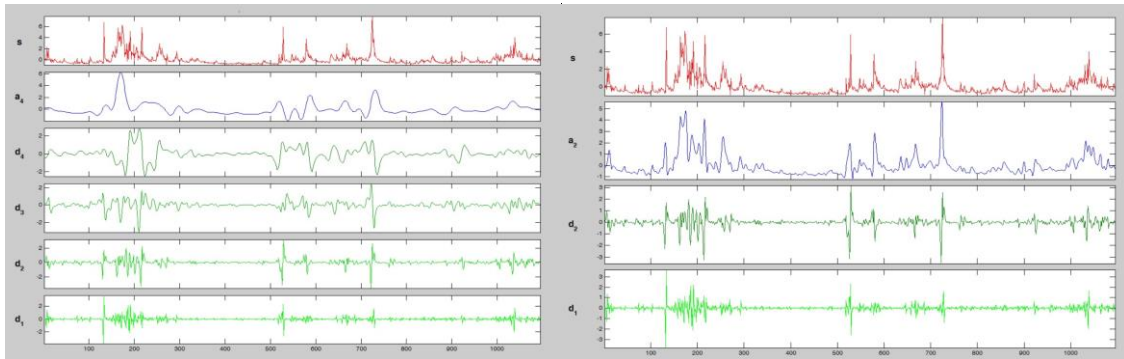


Figure 3: Wavelet decomposed series of level 4 (left) and level 2 (right) [original series (s) in red, approximation in blue and the details in green]

Forecasts for 14 days were obtained for both wavelet based NAR-ANN models and the results were compared with the actual inflow values along with the forecast obtained from single NAR-ANN and cluster based NAR-ANN. The forecasts of the latter two models were obtained according to single NAR-ANN (Basnayake et al., 2017 a) and cluster based NAR-ANN. (Basnayake et al., 2017 b)

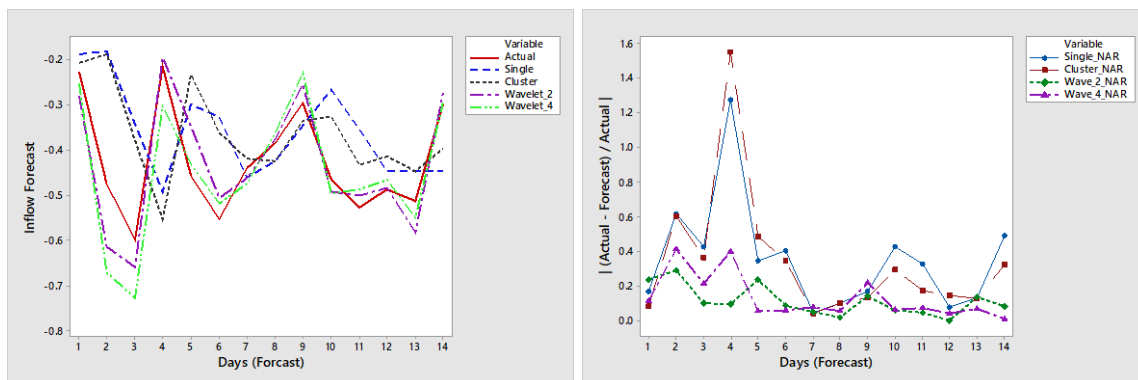


Figure 4: Actual vs. fitted values of 2 weeks ahead forecast of 4 models fitted (left), pointwise errors of the same (right)

As shown in Figure 4, it can be clearly observed that both wavelet based models have outperformed the other approaches at all instances. Pointwise errors with each model were obtained for better comparison of the results. Lowest pointwise errors in general were observed with wavelet NAR-ANN approach with 2 levels. However, there are some instances where the wavelet NAR-ANN with 4 levels produces more accurate results. (With a percentage of 36% where wavelet_4 is better than wavelet_2). The results were analysed in some other parts of the data too and the results were very much similar to above. Even the grid errors as a whole for wavelet based NAR-ANN were lower than that of the other models.

5. CONCLUSION

Wavelet NAR-ANN model has better captured the nonstationary nature of the inflow series when using the approach discrete wavelet decomposition. The divide and conquer approach consisting with 3 decompositions (DWT with 2 levels) has captured the variations in the inflow series very well. Thus the forecast performance of the proposed wavelet NAR-ANN is higher across all the days considered, compared to single NAR-ANN and cluster based NAR-ANN. However, between the two wavelet models compared (3 decompositions and 5 decompositions), around one third (64%) of the time, the model with 3 decompositions has performed better than the one with 5 decompositions. It should be noted that, both models do not show a remarkable difference with respect to the forecast performance. The wavelet NAR-ANN with 2 levels (3 decompositions) is a promising model formulation to obtain

2 weeks ahead inflow forecasts of the Kotmale reservoir in Mahaweli river basin. This paper also illustrates an implementation of the novel approach wavelet NAR-ANN procedure related to daily forecast of a complex non-linear and non-stationary series of data.

6. FUTURE WORK

The favourable performance observed with wavelet NAR-ANN as a univariate approach, can further be investigated by incorporating the exogenous variable rainfall to this approach. Another option for improvement is to adjust for the extreme values and then apply the wavelet NAR-ANN. In addition to this improvements the research can be directed towards applying wavelet based probabilistic forecasting, instead of point forecasts.

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